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Orbit Determination from Minitrack Observations [and Discussion]

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Orbit determination from Minitrack observations

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Though Minitrack observations are only accurate to about 1′, accurate orbits—typically eccentricity to 10^{-5}—have been obtained for a number of satellites. The main sources of observational error, real or apparent, are thought to be: inadequate correction for ionospheric refraction; and inadequate representation of satellite perturbations due to the Earth's tesseral harmonics and to atmospheric drag.

1. Introduction

Radio tracking of satellites has an advantage over radar tracking, in that power requirements are much less, and an advantage over optical observation, in that conditions of light and darkness are irrelevant. The main disadvantage, in comparison with the other two sources of observations, is that there must be a transmitter on board the satellite.

Two possible techniques exist for extracting observational data from radio tracking. They may be employed simultaneously but this is not normal. The first technique—measuring range rate by the Doppler principle—has been described by R. R. Newton (this volume, p. 50). The alternative is to measure direction cosines by the interferometry principle; this is the Minitrack technique.

Though the French have set up two Minitrack stations, and E.S.R.O. are setting up another, the only data so far seen at R.A.E. are from the Stadan network of N.A.S.A. This, presently, consists of a dozen stations as shown in figure 1.

2. The Minitrack principle

If two aerials are set at the ends of a baseline, an observed satellite will, in general, be nearer to one than to the other. The two received signals interfere and the difference in phase gives a direct measurement of the direction cosine of the satellite relative to the baseline. Four aerials, two on a north–south baseline and two on an east–west baseline, will give two direction cosines—that is, a complete specification of direction.

In fact, however, a typical Minitrack station (Hopkins, this volume, p. 46; Truszynski 1961) has 13 aerials. The four-aerial system is duplicated; one system, the 'equatorial system', is much more electrically sensitive in the north–south direction than in the east–west direction; for the other system, the 'polar system', the reverse is true. In addition, the main baselines, being 55 wavelengths (about 120 m) long, lead to phase ambiguities. These ambiguities are resolved by supplementing the two 'fine' systems by 'medium' and 'coarse' systems, requiring five further aerials. Since these aerials are isotropic they serve both the polar and the equatorial systems.

Before a satellite pass, the equatorial or polar system is selected, according to the expected ground-track. This is equivalent to erecting a detection fence through which the satellite, on average, takes about 30 s to pass. The north–south and east–west, fine,

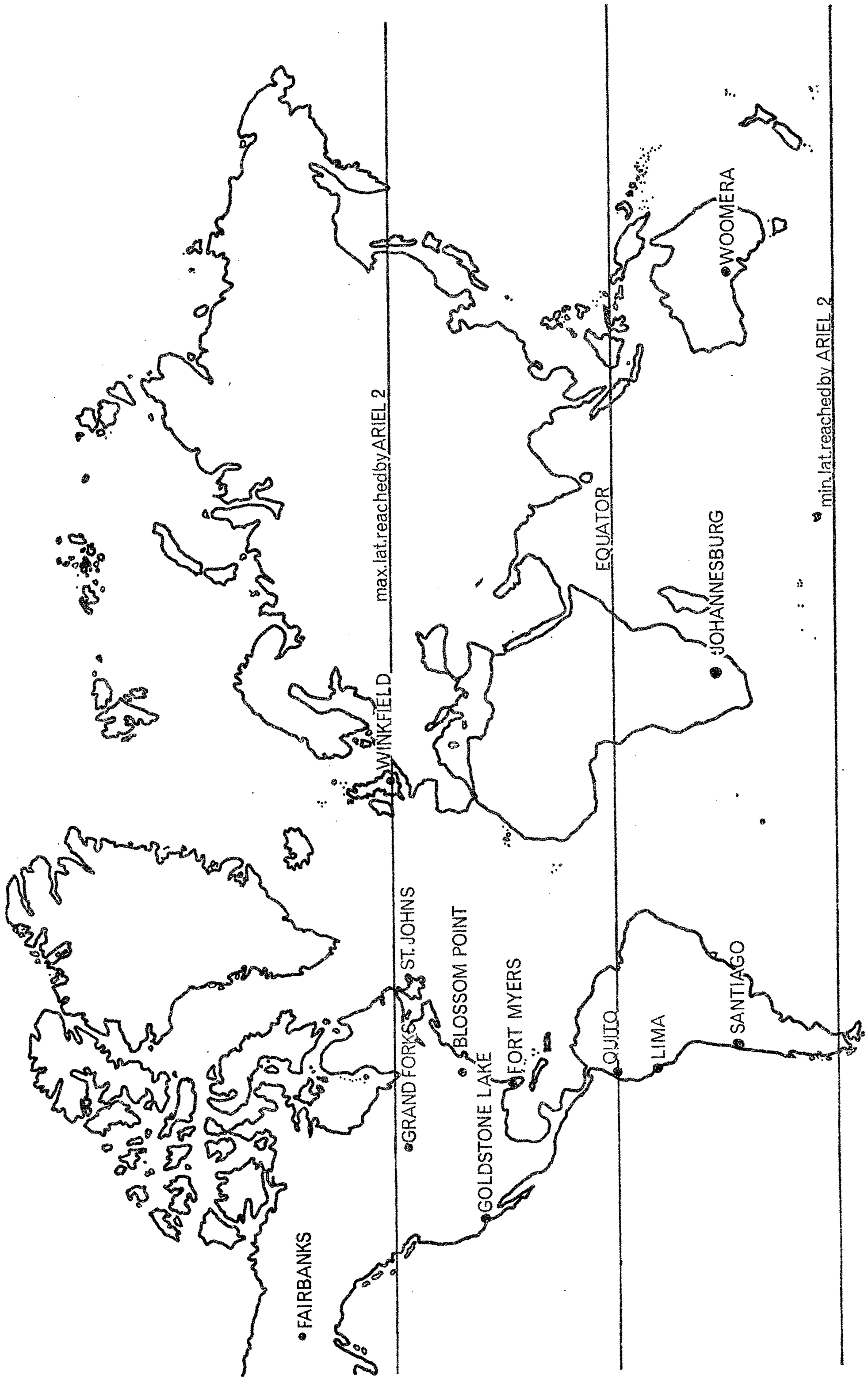

FIGURE 1. The Stadan network of Minitrack stations.

medium and coarse interferometers take 150 measurements of phase difference during this time; these are recorded on paper tape and transmitted to N.A.S.A. (Goddard Space Flight Center).

The raw data from all the stations are processed by N.A.S.A. and reduced to two observations, about 15 s apart, for each station/pass. Corrections for ionospheric refraction are incorporated at this stage. The resulting data, from all satellites currently tracked, are distributed to interested parties, including R.A.E., every few weeks.

3. Orbit determination

Naturally, N.A.S.A. analyse their own data for all satellites tracked. Even when they are not themselves interested in a particular orbit they must do this to provide tracking predictions. Though orbital elements may be obtained from N.A.S.A., they are not formally published. Since 1963 they have been derived using the orbital theory of Brouwer (1959). This means, effectively, that they are derived from a double averaging of osculating elements: first with respect to mean anomaly to remove short-period (J_2) perturbations, and second with respect to argument of perigee, ω, to remove long-period perturbations; long-period terms are removed on the assumption that they are composed of terms in sin ω, cos ω, sin 2ω, etc.; terms in J_2^2, J_3, J_4 and J_5 are accounted for, with, until recently,† the following values taken: $J_2 = 1082{\cdot}19 \times 10^{-6}$, $J_3 = -2{\cdot}285 \times 10^{-6}$, $J_4 = -2{\cdot}123 \times 10^{-6}$. $J_5 = -0{\cdot}232 \times 10^{-6}$.

Orbits for four satellites have been determined at R.A.E. The theory of Merson (1963*a*), in which short-period perturbations are catered for by use of smoothed elements, is the basis of a differential correction programme written for the Pegasus computer (Merson 1963*b*). Long-period perturbations are treated as if secular; over a period of 3 or 4 days this does not involve any noticeable loss in accuracy.

4. Orbits of Tiros 7 and Alouette 1

The orbit of Tiros 7 (1963–24A) has been determined, during a 10-day period, by Scott (1965), and the orbit of Alouette 1 (1962 $\beta\alpha 1$), during an 8-day period, by Hiller (1966). The orbital inclinations are, respectively, 58° and 80°. The small-eccentricity theory of Cook (1966) has been used in interpreting the orbits, and orbital elements have been compared with values provided by N.A.S.A. The eccentricity, e, was less than 0·005 for both satellites and it was decided to analyse the Tiros orbit just after a maximum in an eccentricity cycle—eccentricity being maximum when ω, the argument of perigee, was 90°—and the Alouette orbit around a minimum in the cycle, i.e. near $\omega = 270°$. Differences between R.A.E. and N.A.S.A. elements were expected to be largest at eccentricity maxima and minima.

Results showed that the small-eccentricity theory, in which elements e and ω are replaced by $e \cos \omega$ and $e \sin \omega$, describes this type of orbit much better than the ordinary sinusoidal theory of long-period motion. This is illustrated in figure 2 which shows how well the theoretical curve, drawn over a complete cycle, fits the observed values of eccentricity

† Currently the values used are $1082{\cdot}48 \times 10^{-6}$, $-2{\cdot}56 \times 10^{-6}$, $-1{\cdot}84 \times 10^{-6}$ and $-0{\cdot}06 \times 10^{-6}$.

for Alouette 1. A sinusoidal curve gives only an approximate fit. N.A.S.A. values of eccentricity, shown over a complete cycle of perigee, were presumably obtained by subtracting a sinusoidal term from the normal eccentricity, the amplitude of this term (about 0·8 of the amplitude of the dashed curve in figure 2) being given by the N.A.S.A. values of J_3 and J_5 already quoted—the contribution from J_5 is actually negligible. Correction of the N.A.S.A. eccentricities on this basis, however, gives values which still appear to be inconsistent with the theoretical behaviour.

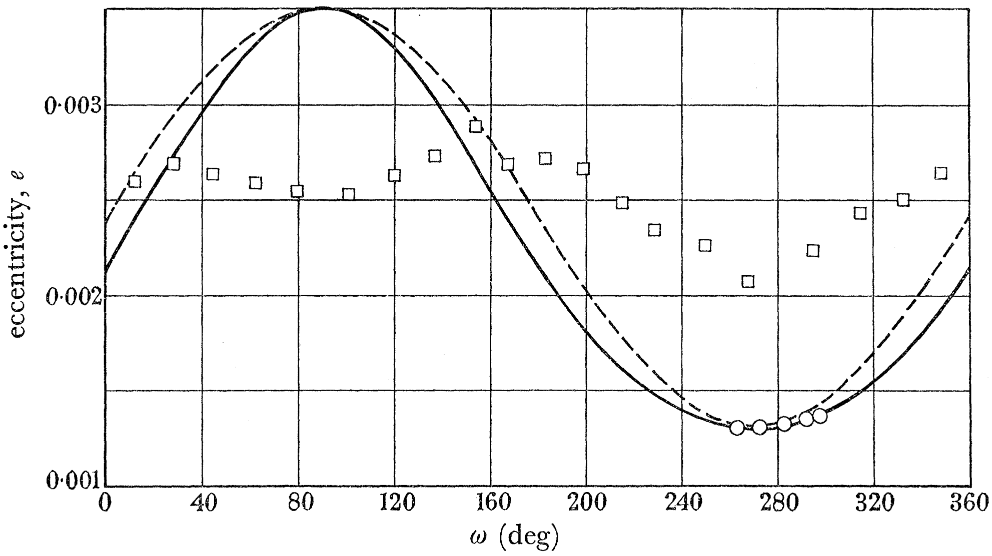

FIGURE 2. Eccentricity of Alouette 1 plotted against argument of perigee. □, N.A.S.A. values; ○, R.A.E. values; – – –, sinusoidal variation; ——, variation given by small-eccentricity theory.

Average accuracies (s.d.'s) for computed orbital parameters are listed in table 1. Figures for Ariel 2, discussed in the next section, are included. The parameters n_1 and n_2 are linear and quadratic coefficients in the mean-motion polynomial.

TABLE 1. ACCURACIES OF COMPUTED ORBITAL ELEMENTS

	Tiros 7	Alouette 1	Ariel 2
semi-major axis, a	0·4 m	1 m	1 m
eccentricity, e	6×10^{-6}	14×10^{-6}	8×10^{-6}
inclination, i	0·0006°	0·0012°	0·0005°
R.A. of node, Ω	0·0008°	0·0013°	0·001°
argument of perigee, ω†	0·0005°	0·0012°	0·0005°
time at node, t_0	20 ms	25 ms	30 ms
n_1 (deg/(100 days)^2)	—	17	12
n_2 (deg/(100 days)^3)	—	—	1500

† For meaningful comparison with other elements, the s.d.'s of ω have been multiplied by e.

Since both Tiros 7 and Alouette 1 were in virtually drag-free orbits, Tiros 7 being at a height of 635 km and Alouette 1 at a height of 1000 km, the very low s.d. for semi-major axis really does provide a valid estimate of random error for these two satellites, though it does not for Ariel 2 which is much more affected by drag (Gooding 1966). There may, however, be a systematic error as large as 20 m, arising from error in the assumed value (398 602 km^3/s^2) of the earth-mass × gravitation constant.

5. Orbits of Ariel 1 and Ariel 2

The orbit of Ariel 1 (1962 o1) was the first to be determined at R.A.E. from Minitrack data (when the computer program was still under development). The object in determining this orbit was to assess the accuracy that was likely to be attainable for the satellite U.K. 2 which had not then been launched. Whereas N.A.S.A. provided the experimenters with definitive elements for Ariel 1, it was intended that R.A.E. should do this for U.K.2.

U.K. 2 was launched on 27 March 1964 and was then rechristened Ariel 2 (1964–15A). The orbital inclination is 51·6°; initial perigee and apogee heights were 285 and 1360 km. The orbit was determined every $1\frac{3}{4}$ days (at 25 node intervals, to be exact) for a period of 12 months; satellite position can be computed, using the elements listed by Gooding (1966),

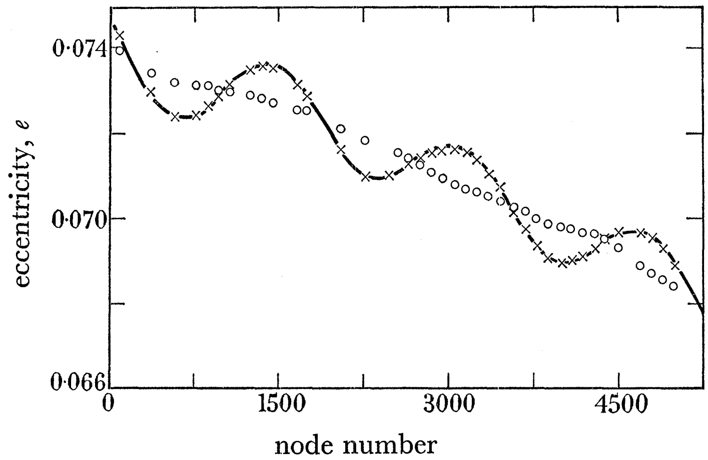

Figure 3. Eccentricity of Ariel 2: N.A.S.A. and R.A.E. values compared. ——, R.A.E. curve; ○, N.A.S.A. values; ×, N.A.S.A. values corrected.

for any instant during this period, with the error never exceeding 1 km or so. Average s.d.'s have been listed in table 1; inclination is determined so accurately (to about 0·0005° in s.d.) that the change in i over a year has been used by King-Hele & Scott (1966) in a study of the rotation of the upper atmosphere, even though this change was much smaller (about 0·005°) than the corresponding change for satellites in lower orbits. The amplitude of the oscillation in the sinusoidal component of the eccentricity variation has been used by King-Hele, Cook & Scott (this volume, p. 144), in conjunction with data from other satellites, to obtain values for the Earth's odd zonal harmonics, J_3, J_5, etc. The rate of change of Ω has been used, similarly, by Smith (1965) in an evaluation of the even harmonics.

A plot of eccentricity over 12 months is given in figure 3. The weekly values of N.A.S.A. are also plotted; the long-period oscillation, of amplitude 0·001, has clearly been removed. On restoring this oscillation, the corrected values of N.A.S.A. lie on the R.A.E. curve. It is useful to remove the secular trend and part of the oscillation from the eccentricity, in order to exhibit its behaviour on a ten-times-larger scale. Figure 4 shows the result; values are represented by lines of length 2 s.d.'s, the average value of an s.d. being 0·00001.

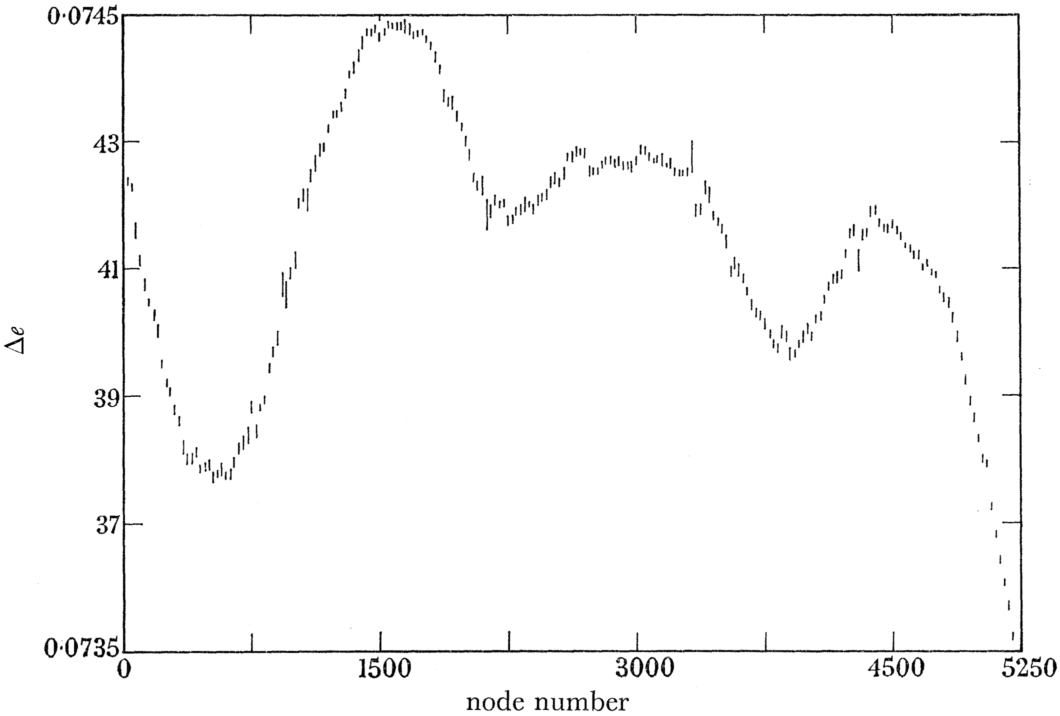

FIGURE 4. Eccentricity of Ariel 2: linear and sine terms removed.

6. An unexplained discrepancy

To give the Ariel 2 experimenters their requested accuracy—satellite position to $\frac{1}{2}$ km whenever possible—it was found necessary to remove an apparent along-track error in satellite computed position. This error was sinusoidal, with amplitude 650 m and period just under half a day. It was diagnosed as due to the effect of certain of the tesseral harmonics of the Earth's gravitational field; due in fact to those coefficients $J_{n,m}$ for which $m = 2$ and $n(\geqslant m)$ is even.

Now approximate values of $J_{2,2}$, $J_{4,2}$, $J_{6,2}$ and $J_{8,2}$ are available. They have been computed by fitting to observed perturbations for a number of satellites, though not for any with orbits determined from Minitrack observations. These values predict an along-track oscillation for Ariel 2, of the period observed but of amplitude only half that observed. Expressing the along-track oscillation as a time error in fact (where $7\frac{1}{2}$ km are equivalent to 1s), the amplitudes contributed by Guier & Newton's (1965) values of $J_{2,2}$, $J_{4,2}$, $J_{6,2}$ and $J_{8,2}$ should be respectively 0·051, 0·017, 0·004 and 0·002 s, giving (allowing for the different phases) a total of 0·051 s. The observed amplitude, however, is 0·088 s, as shown in figure 5.

Thus there is a discrepancy of which the explanation is not known. The Guier–Newton value of $J_{2,2}$ has been confirmed, to within a few parts per cent, by Allan & Piggott (this volume, p. 137) using results from the synchronous satellites Syncom 2 and Syncom 3, and it would be inconceivable that this value should be wrong by a factor of nearly 2. It is interesting to note, however, that if we *do* make this assumption, the observed along-track oscillation is obtained not only for Ariel 2 but also for the satellite Ogo 2 (1965–81A) on which some preliminary analysis, using Minitrack observations, has been done. The orbital inclination of this satellite is very different (87·4°) so that the effects of $J_{2,2}$, $J_{4,2}$, etc., should combine differently for the two satellites.

Several possible explanations have been suggested, but each has had to be rejected. The most likely is still that the true values of $J_{4,2}$, $J_{6,2}$, etc., are much bigger than those in Guier & Newton (1965). The author is not aware of any other perturbation which would be of the right period, namely $\pi/(\dot{\theta}-\dot{\Omega})$, where θ is the sidereal time and Ω is the right ascension of the satellite node; for Ariel 2 it is known very accurately that the observed perturbation has this period, since the orbit has been analysed for a full year.

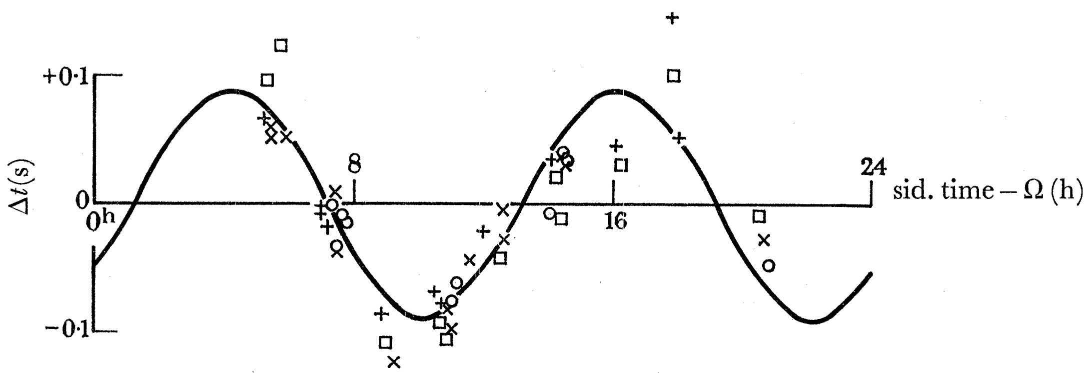

FIGURE 5. Apparent time error (Ariel 2).

7. Accuracy of Minitrack observations

Although the quantities observed by the Minitrack stations are direction cosines, these are converted (in R.A.E. analysis) into azimuth and elevation before use. The differential correction programme works on the assumption that the errors in all observations are random, uncorrelated and normally distributed; furthermore, that

$$\cos E \times \sigma(A) = \sigma(E) = 1',$$

where A and E denote azimuth and elevation; time errors are ignored. The standard deviation assigned (1′) is a round figure based on experience. At the end of every complete orbit determination an *a posteriori* estimate of accuracy is computed and included, as a factor, in the estimated standard deviations of the orbital elements. This *a posteriori* estimate is normally between 1′ and 2′.

The estimated instrumental accuracy of the Minitrack system, when it was designed, was about 20″, the unit of resolution being about 5″ (corresponding to 0·36 electrical degree in raw phase measurement). After smoothing, the random error should be negligible, so that it is of interest to know why the *a posteriori* accuracy of an observation is as bad as it is—the best visual observers can equal this performance of 1 or 2′.

Error arises from the following sources: phase drift, ionospheric refraction, station survey, timing; apparent error (as shown by residuals) will also arise from deficiencies in the orbital model used in the analysis.

Phase drift may build up to 30″ over a 6-month period, after which there is a recalibration. Errors from ionospheric refraction are liable to be much larger than this.

However, large errors are avoided by N.A.S.A. who (*a*) correct for ionospheric refraction (using ionospheric predictions from I.T.S.A., Boulder), and (*b*) normally restrict disseminated data to observations when the elevation is greater than 70°.

The stations have recently been surveyed relative to the Fischer ellipsoid and errors in their position should not exceed 100 m. Timing—local timing, at any rate—is thought to be good to a millisecond or so.

Thus, of the genuine observational errors, only those due to ionospheric refraction are likely to be large enough to account for the residuals obtained. The author would have thought that even these, in general, would not be large enough.

Two deficiencies in the model must be mentioned. The first concerns the tesseral harmonics of the Earth's gravitational field. As has been explained, allowance is made for an along-track perturbation of $\frac{1}{2}$ day period. Apart from this, however, the tesseral harmonics have so far been neglected. Typically this may lead to residuals of 1′, or more for close satellites, but no pattern in the residuals has been detected, of the type which would indicate that this is the dominant cause of (apparent) observational error.

The other deficiency concerns the representation of drag perturbations. This is inevitable, unlike the situation for tesseral harmonic effects which will eventually be programmed, since the atmosphere fluctuates from hour to hour. For a satellite like Ariel 2, though a cubic polynomial is fitted to the mean anomaly to represent the perturbation as far as possible, the residual effects could be large enough to account for the observed residuals. For satellites with much higher perigees, like Tiros 7 and Alouette 1, the residual drag effects should be small; the *a posteriori* estimate of accuracy for these satellites is still (Scott 1965; Hiller 1966) about 1·3′, however.

8. Accuracy of a minimal network

Since E.S.R.O., or the U.K. if a national programme is undertaken, will not normally have access to the Stadan network, it is important to know how the accuracy of orbit determination is affected when only two or three stations are providing observations. Some accuracy assessment studies along these lines have been carried out at R.A.E.

Consider, as an example, the case of a polar orbit with perigee height 300 km and apogee height 1000 km, with the satellite observed by two stations, one in south-eastern England or Belgium and the other in Southern Australia. Merson (1966) has shown that at each station there should be an average of about one pass per day for which the satellite reaches an elevation greater than 50°. Over 3 days this would give six passes, i.e. twelve observations, assuming two per pass. For a particular choice of longitude crossing at the central node, an accuracy assessment was made; with this choice there was only one pass from the first station, but five for the second. Assuming an accuracy of $1\frac{1}{2}'$ for the observations and a seven-parameter model, accuracies (s.d.'s) of orbital parameters worked out as follows: $\sigma(a) = 4$ m, $\sigma(e) = 10^{-5}$, $\sigma(i) = \sigma(\Omega) = 0{\cdot}0015°$, $e\sigma(\omega) = 0{\cdot}001°$, $\sigma(t_0) = 80$ ms and $\sigma(n_1) = 40$ deg $(100 \text{ days})^{-2}$. With these accuracies a satellite ephemeris to an accuracy of 1 km should normally be attainable.

As a practical example of what can be achieved, the orbit of Ariel 2, over the first $3\frac{1}{2}$ days after launch, has been re-analysed, using observations from Winkfield (England)

and Johannesburg only. With twelve observations—all at elevation greater than 50°—and a seven-parameter model (no n_2) the following s.d.'s were obtained: $\sigma(a) = \frac{1}{2}$ m (largely fictitious due to drag fluctuations), $\sigma(e) = 10^{-5}$, $\sigma(i) = 0{\cdot}0007°$, $\sigma(\Omega) = 0{\cdot}0017°$, $e \times \sigma(\omega) = 0{\cdot}0007°$, $\sigma(t_0) = 54$ ms and $\sigma(n_1) = 12$ deg (100 days)$^{-2}$. Estimated ephemeris errors were less than $\frac{1}{2}$ km, though it is thought that errors would be larger than this due to biases in the observations.

As an exercise in what is probably the ultimate in reducing the network, the example just quoted was repeated, with seven observations, from Johannesburg only, using a six-parameter model (neither n_1 nor n_2). The following were the s.d.'s: $\sigma(a) = 1\frac{1}{2}$ m, $\sigma(e) = 10^{-4}$, $\sigma(i) = 0{\cdot}005°$, $\sigma(\Omega) = 0{\cdot}003°$, $e \times \sigma(\omega) = 0{\cdot}005°$ and $\sigma(t_0) = 0{\cdot}14$ s. Estimated ephemeris errors rose to 4 km.

This last example is somewhat exceptional, as will now be seen. There are two complementary requirements in order that orbital analysis shall yield good orbital parameters. The first is good coverage in *time*. Here there is usually no difficulty. To obtain a good value of mean motion (equivalent to semi-major axis) it is desirable to have two passes $3\frac{1}{2}$ days apart, say, with the same part of the orbit observed during each pass. (A third pass, half way between, will allow a value of n_1 to be computed, etc.) For the other elements, however, it is essential to have a good coverage in *angle* (in true anomaly, say) and this is the second requirement. This is rarely available from one station, but in the example quoted Johannesburg observed one south-going pass as well as a number of north-going passes. The seven observations used consisted, in fact, of two from a north-going pass, three from a south-going pass $1\frac{1}{2}$ days later, and then two from a north-going pass $1\frac{1}{2}$ days after that.

9. Conclusions

Orbits of a number of satellites have been analysed at R.A.E., using Minitrack data kindly supplied by N.A.S.A. Results have been very satisfactory, with average accuracies (s.d.) including 10^{-5} in eccentricity and $0{\cdot}001°$ in inclination and right ascension of the node. These accuracies are due to the excellent global coverage of the Stadan Minitrack network and to the fact that radio observations are not hampered by visibility restrictions.

The accuracy of the observations themselves is disappointing. At 1 or 2 min of arc it is no better than can be achieved by the best visual observers. Errors, real or apparent, may be attributed to a number of sources: inadequate correction for ionospheric refraction, irregularities in atmospheric drag, incomplete representation of tesseral harmonics, phase drift, station survey and timing. The main sources are likely to be the first three of these, but it should be possible to reduce the effects, even of these, below the 1′ level.

To obtain well-fitting orbits for the satellite Ariel 2, it has been found necessary to use an unnaturally high value of the tesseral harmonic $J_{2,2}$. The reason for this is not yet known.

I am grateful to Dr J. W. Siry of N.A.S.A. for some helpful comments.

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Discussion

Dr J. W. Siry (N.A.S.A. Goddard Space Flight Center, Greenbelt, Maryland): Mr Gooding has pointed out that the overall accuracy of a Minitrack observation is not as good as the instrumental accuracy. The ionospheric refraction corrections are based on predictions made three months in advance by the Institute for Telecommunications Sciences and Aeronomy. These are not nearly so good as the corrections applied in two-frequency systems, such as certain Doppler systems, where each set of tracking data includes measures of the corresponding ionospheric refraction. I suspect that the principal source of error in the Minitrack observations is the lack of an accurate ionospheric refraction correction.

I should like to emphasize that the 5″ mentioned by Mr Gooding is a precision or resolution capability: the design figure for the instrumental accuracy of Minitrack excluding ionospheric effects was 20″.

In reply to a question, Dr Siry remarked that the ionospheric refraction correction could be as much as 3′.